ALGEBRA.

113. Proposd by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find a square number consisting of 24 figures such that the numbers formed by the first 12 figures and the last 12 figures, respectively, are consecutive, and *vice versa*.

Solution by the PROPOSER.

(1). Let $y^2=1000000000001x+1000000000000$ be the number.

$\therefore 1000000000001x=y^2-1000000000000$.

$\therefore x=\dfrac{(y+1000000)(y-1000000)}{10001\times 99990001}$.

Let $y+1000000=99990001u$, $y-1000000=10001v$.

$\therefore 10001v=99990001u-2000000$. $v=9998u-199+\dfrac{3u-9801}{10001}$.

Let $S=\dfrac{3u-9801}{10001}$. $\therefore u=\dfrac{10001S+9801}{3}=3333S+3267+\dfrac{2S}{3}$.

Let $S=3m$. $\therefore u=10001m+3267$.

$y=1000000000001m+326666333267$.

$x=(99990001m+32663267)(10001m+3267)$.

Let $m=0$. $\therefore y=326666333267$, $x=106710893289$.

$y^2=106710893290106710893289=1000000000000(x+1)+x$.

(2). $y^2=1000000000001x+1$, or $x=\dfrac{(y+1)(y-1)}{(10001)(99990001)}$.

$y+1=99990001u$, $y-1=10001v$.

$\therefore v=\dfrac{9998u+(3u-2)}{10001}=9998u+S$. $\therefore u=\dfrac{3333S+(2S+2)}{3}=3333S+t$.

$\therefore S=t-1+\frac{1}{2}t=t-1+m$. $\therefore S=3m-1$, $u=10001m-3333$.

$y=1000000000001m-333266673334$.

$x=(10001m-3333)(99990001m-33323335)$.

Let $m=1$.

$\therefore y=666733326667$, $x=444533328888$.

$y^2=444533328888444533328889=1000000000000x+(x+1)$.

114. Proposed by ELMER SCHUYLER, B. Sc., Professor of German and Mathematics in Boys' High School, Reading, Pa.

$$\left.\begin{aligned} bx^3&=10a^2bx+3a^3y \\ ay^3&=10b^2ay+3b^3x \end{aligned}\right\}$$

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and the PROPOSER.

$bx^3=10a^2bx+3a^3y$......(1), $ay^3=10ab^2y+3b^3x$......(2).

Let $x/a=u$, $y/b=v$, then (1) and (2) become

$u^3=10u+3v$......(3), $v^3=10v+u$......(4).

(3)+(4) gives $u^3+v^3-13(u+v)=0$......(5).

(3)−(4) gives $u^3-v^3-7(u-v)=0$......(6).

From (5), $u+v=0$......(7); $u^2-uv+v^2=13$...... (8).

From (6), $u-v=0$......(9), $u^2+uv+v^2=7$......(10).

From (7) and (9), $u=0$, $v=0$.

(8)+(10) gives $u^2+v^2=10$ .....(11).

(8)−(10) gives $uv=-3$......(12).

From (11) and (12), $u+v=\pm2$, $u-v=\pm4$.

$\therefore$ $u=\pm3$ or ±1, $v\mp1$ or ∓3.

From (7) and (10), $u=\pm\sqrt{7}$, $v=\mp\sqrt{7}$.

From (9) and (8), $u=\pm\sqrt{13}$, $v=\pm\sqrt{13}$.

$\therefore$ $u=0,\ 1,\ -1,\ 3,\ -3,\ \sqrt{7},\ -\sqrt{7},\ \sqrt{13},\ -\sqrt{13}$.

$v=0,\ -3,\ 3,\ -1,\ 1,\ -\sqrt{7},\ \sqrt{7},\ \sqrt{13},\ -\sqrt{13}$.

$x=0,\ a,\ -a,\ 3a,\ -3a,\ a\sqrt{7},\ -a\sqrt{7},\ a\sqrt{13},\ -a\sqrt{13}$.

$y=0,\ -3b,\ 3b,\ -b,\ b,\ -b\sqrt{7},\ b\sqrt{7},\ b\sqrt{13},\ -b\sqrt{13}$.

II. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Let $bx/ay=t$.

Divide (1) by (2), then $\dfrac{b^3x^3}{a^3y^3}=\dfrac{10bx+3ay}{10ay+3bx}$, or $t^3=\dfrac{10t+3}{10+3t}$.

Whence, $3t^4+10t^3-10t-3=0$.

Factoring, $(t^2-1)(3t^2+10t+3)=0$.

Whence, $t=1,\ -1,\ -\frac{1}{3},\ -3$.

When $t=1$, $bx=ay$ and substituting in (1) we easily get,

$$x=0, \text{ or } \pm a\sqrt{13}.$$
$$y=0, \text{ or } \pm b\sqrt{13}.$$

When $t=-1$, $bx=-ay$. and we get in a similar manner,

$$x=0, \text{ or } \pm a\sqrt{7}.$$
$$y=0, \text{ or } \mp b\sqrt{7}.$$

When $t=-\frac{1}{3}$, $\quad x=0$, or $\pm3a$.

$$y=0, \text{ or } \mp b.$$

When $t=-3$, $x=0$, or $\pm a$.
$y=0$, or $\mp 3b$.

Also solved by *J. M. BOORMAN*, and *HARRY S. VANDIVER*.

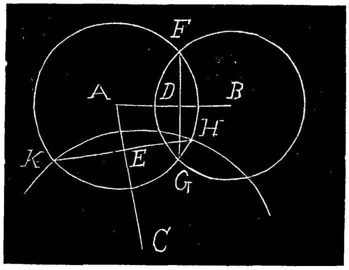
GEOMETRY.

143. Proposed by J. T. FAIRCHILD, A. M., Instructor in Mathematics, Crawfis College, Crawfis College, Ohio.

If the centers of three spheres do not lie in the same straight line, their surfaces cannot have more than two points in common. These points lie in a straight line perpendicular to the plane of centers and equal distances from this plane on opposite sides. [From *Phillips and Fisher's Geometry*.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let A, B, C be the centers of the three spheres, respectively. B intersects A in the circle, radius FD, such that FD is perpendicular to AB and D lies on AB. C intersects A in the circle, radius EH, such that EH is perpendicular to AC and E lies on AC. The two circles D and E on the same sphere A can intersect in only two points situated on a line perpendicular to the plane AED and equally distant from it. But plane AED coincides with plane ABC. Therefore, the truth of the theorem follows.



144. Proposed by L. C. WALKER, Assistant in Mathematics, Leland Stanford, Jr. University, Palo Alto, Cal.

Find the equations of four cones that pass through three given straight lines intersecting in the same point.

I. Solution by the PROPOSER.

Let the mutual inclination of the line be 2α, 2β, 2γ, and let the equation of the cone be referred to the three given straight lines as coördinate axes.

The equation of the concentric sphere referred to the same axes is

$$x^2+y^2+z^2+2yz\cos2\alpha+2zx\cos2\beta+2xy\cos2\gamma=r \quad \ldots(1).$$

The equation of the plane that passes through the intersection of the cone and (1) is

$$\frac{x}{r}+\frac{y}{r}+\frac{z}{r}=1 \ldots(2).$$

Now making (1) homogeneous by means of (2), and reducing, we have for the equation of the cone,